TEST YOUR SELF - 2

Mathematics - Class – XII

Time: 3 hr. Max. Marks: 100

General Instructions

1. *All questions are compulsory.*
2. *The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.*
3. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
4. *There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six mark each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculators is not permitted. You may ask for logarithmic tables, if required.*

SECTION - A

1. Find *fog*, if $f(x) = |x|$ and $g(x) = cosx$
2. What is the value of $\tan\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$
3. A is a square matrix. Write the value of A(adj A)
4. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then find the value of x
5. What is the maximum value of $\left(\frac{1}{x}\right)^x$
6. Write a value of $\int_0^2 x[x]dx$
7. If A is a non singular matrix of third order then write the equivalents to the following:
   a. A. Adj(A)
   b. If $|A| = k$, then $|5A| =$ ________
   c. $(A^T)^{-1} =$ __________
8. What is the degree of the equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} = x$
9. Find the area of the parallelogram, whose diagonals are along the vectors $\hat{i} + 2\hat{k}$ and $2\hat{j} - 3\hat{k}$
10. Write the direction cosines of a unit vector perpendicular to the plane $\vec{r}.(6\hat{i} - 3\hat{j} - 2\hat{k}) - 1 = 0$

SECTION - B

11. Let * be a binary operation defined on N × N, by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative and associative. Also find the identity element for * on N × N, if any.

(Or)
Consider $f: R \to [-5, \infty[$ given by $f(x) = 9x^2 + 6x - 5$. Show that f invertible and find its inverse.

12. If $\cos^{-1}a + \cos^{-1}b + \cos^{-1}c = \pi$, prove that $a^2 + b^2 + c^2 + 2abc = 1$

13. Using elementary operation, find the inverse of matrix A = $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$

OR

Using properties of determinants, prove that

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

14. Find all the points of discontinuity of the function f defined by

$$f(x) = \begin{cases} x+2, x<1 \\ 0, \quad x=1 \\ x-2, x>1 \end{cases}$$

15. If y = $\sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \ldots\infty}}}$, prove that $\frac{dy}{dx} = \frac{\sec^2 x}{(2y-1)}$

OR

If $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$, find $\frac{dy}{dx}$

16. Evaluate: $\int \frac{1}{x^4+1} dx$

17. Show that a closed right circular cylinder of given total surface area and maximum volume is such that its height is equal to the diameter of its base

18. Evaluate : $\int_0^2 (2x+3)dx$,as a limit of sums

19. Using properties of definite integrals, evaluate : $\int_0^{\pi/2} \frac{\sin^7 x}{\sin^7 x + \cos^7 x} dx$

20. Find the foot of the perpendicular drawn from the points A(1, 0, 3) to the join of the points B(4, 7, 1) and C(3, 5, 3)

21. Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

22. A bag X contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bag and is found to be red. Find the probability that it was drawn from bag Y

OR

A candidate has to reach the examination centre in time. Probability of him going by bus or scooter or by other means of transport is $\frac{3}{10}, \frac{1}{10}, \frac{3}{5}$ respectively. The probability that he will be late is $\frac{1}{4}$ and $\frac{1}{3}$ respectively, if he travels by bus or scooter. But he reaches in time if he uses any other mode of transport. He reached late at the centre. Find the probability that he traveled by bus.

SECTION - C

23. Find the matrix X such that, $\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix} X = \begin{bmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}$

24. Find all the local maximum values and local minimum values of the function $f(x) = \sin 2x - x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$

OR

A given quantity of metal is to be cast into a solid half circular cylinder (i.e., with rectangular base and semicircular ends). Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its circular ends is

25. Draw the rough sketch of y = sin 2x and determine the area enclosed by the curve, the x-axis and the lines $x = \pi/4$ and $x = 3\pi/4$

26. If A= $\begin{bmatrix} 2 & 1 & 3 \\ 3 & -1 & 4 \\ 2 & 3 & 5 \end{bmatrix}$ Find A^{-1}. Using A^{-1}, solve the following system of linear equations

$$2x+3y+2z=3$$
$$x-y+3z=8$$
$$3x+4y+5z=9$$

27. Two bags A and B contain 4 white 3 black balls and 2 white and 2 black balls respectively. From bag A two balls are transferred to bag B. Find the probability of drawing
    (a) 2 white balls from bag B?
    (b) 2 black balls from bag B?
    (c) 1 white & 1 black ball from bag B?

28. Anil wants to invest at Rs 12,000 in Bonds A and B. According to the rules, he has to invest at least Rs. 2,000 in Bond A and at least Rs 4,000 in Bond B. If the rate of interest on Bond A is 8% per annum and on Bond B is 10% per annum, how should he invest his money for maximum interest? Solve the problem graphically.