

MATHEMATICS - CLASS XII

TIME: 3 hrs

Max. Marks: 100

Section A: Q1 to Q10 each question carries 1 mark

Section B: Q11 to Q22 each question carries 4 marks

Section C: Q23 to Q29 each question carries 6 marks

SECTION - A

1. Give an example of a relation which is reflexive and symmetric but not transitive.

2. Find the value of $\tan^{-1} \left(\frac{-1}{2} \right) + \cos^{-1} \left(\frac{-1}{2} \right) + \sin^{-1} \left(\frac{-1}{2} \right)$

3. Show that the matrix $\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 5 \\ 3 & -5 & 0 \end{bmatrix}$ is a skew symmetric matrix.

4. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

5. Write the cofactors of the elements of the third row of the determinant

$$\Delta = \begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$$

6. Show that $f(x) = \cot^{-1} x$ is always a decreasing function.

7. Find $\frac{dy}{dx}$ if $x^5 + y^5 + 5xy = 100$

8. The Cartesian equations of a line are $6x - 2 = 3y + 1 = 2z - 2$.
Find its direction cosines.

9. Find the position vector of the point which divides the join of the points $2\vec{a} - 3\vec{b}$ and $3\vec{a} - 2\vec{b}$ externally in the ratio 2:3.

10. Evaluate $\int \frac{\sin(x-a)}{\sin x} dx$

SECTION-B

11. Solve the following equations simultaneously:

$$\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3} \quad \text{and} \quad \cos^{-1} x - \cos^{-1} y = \frac{\pi}{3}$$

12.
$$\int \frac{\sqrt{x^2+1} \left[\log(x^2+1) - 2 \log x \right]}{x^4} dx$$

13. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

- (i) none
 - (ii) not more than one
 - (iii) more than one
 - (iv) at least one
- Will fuse after 150 days of use.

14. Evaluate using limit as sum $\int_0^1 e^{2-3x} dx$.

15. Consider a binary operation \oplus on the set $\{1, 2, 3, 4, 5\}$ given by the following multiplication table:

\oplus	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

- (i) Compute $(3 \oplus 4)$ and $(2 \oplus 4)$
- (ii) Compute $(3 \oplus 5)$
- (iii) Is \oplus commutative?

16. Form the differential equation representing the family of ellipses having foci on x-axis and centre at the origin.

17. Using properties of determinants show that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

18. If $y = \left(x + \frac{1}{x}\right)^x + x^{\left(1 + \frac{1}{x}\right)}$, find $\frac{dy}{dx}$

19. Find the intervals in which the function

$f(x) = \log(x+1) - \frac{x}{x+1}$ is increasing or decreasing.

20. Solve the differential equation $\left[x \sin^2\left(\frac{y}{x}\right) - y \right] dx + x dy = 0$; $y = \frac{\pi}{4}$ when $x = 1$

21. Find a vector of magnitude 11 units which is perpendicular to both the vectors $4\hat{i} - \hat{j} - 8\hat{k}$ and $-\hat{j} + \hat{k}$.

22. Find the foot of the perpendicular from the point $(3, -1, 11)$ on the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also find the length of the perpendicular.

SECTION- C

Q23. Using integration find the area of the region given below:

$$x, y : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2$$

Q24. Find the shortest distance between the lines

$$\frac{x+1}{7} = \frac{y+1}{6} = \frac{z+1}{1} \quad \text{and} \quad \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Q25. Given three identical boxes 1, 2 and 3, each containing two coins. In box 1, both coins are gold coins, in box 2, both are silver coins and in box 3, there is 1 gold and 1 silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?

Q26. Evaluate $\int_0^{\pi} \log(1 + \cos x) dx$

Q27. Show that the maximum Volume of a cylinder which can be inscribed in a cone of height

$$h \text{ and semi vertical angle } \alpha \text{ is } \frac{4}{27} \pi h^3 \tan^2 \alpha.$$

Q28. Using elementary transformations, find the inverse of the matrix $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

Q29. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs.600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?