

PRACTICE DIFFERENTIAL EQUATIONS

- 1) Write the order and degree of the differential equation. $[1 + (y')^2]^{3/2} = y''$
(Ans: 2,2)
- 2) Form the differential equation of the family of curves represented by the equation
 $(x+a)^2 - 2y^2 = a^2$ (Ans: $x^2 + 2y^2 - 4xyy' = 0$)
- 3) Write the order and degree of the diff. equation $\left(\frac{d^2y}{dx^2}\right)^{2/3} = \left(y + \frac{dy}{dx}\right)^{1/2}$
(Ans: 2,4)
- 4) Form the differential equation of circles represented by
 $(x-\alpha)^2 + (y-\beta)^2 = r^2$ by eliminating α and β Ans: $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = r^2 \left(\frac{d^2y}{dx^2}\right)^2$
- 5) Show that $Ax^2 + By^2 = 1$ is a solution of the diff. equation. $x \left[y \left(\frac{d^2y}{dx^2} \right) + \left(\frac{dy}{dx} \right)^2 \right] = y \frac{dy}{dx}$
- 6) Solve $(x+y+1) \frac{dy}{dx} = 1$ [Ans: $x = ce^y - y - 2$]
- 7) Solve: $(x+y)^2 \frac{dy}{dx} = a^2$ [Ans: $(x+y) - a \tan^{-1} \left(\frac{x+y}{a} \right) = x + c$]
- 8) Solve: $x \frac{dy}{dx} = y(\log y - \log x + 1)$ [Ans: $y = xe^{cx}$]
- 9) Solve: $ydx - (x+2y^2)dy = 0$ [Ans: $\frac{x}{y} = 2y + c$]
- 10) Solve: $[x\sqrt{x^2+y^2} - y^2]dx + xydy = 0$ [Ans: $\sqrt{x^2+y^2} + \log \frac{y}{x} = Cx$]
- 11) Solve: $\frac{dy}{dx} = \cos^3 x \sin^4 x + x\sqrt{2x+1}$ [Ans: $y = \frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + \frac{1}{10} (2x+1)^{5/2} - \frac{1}{6} (2x+1)^{3/2} + c$]
- 12) Solve: $(x^3y^3 + xy) \frac{dy}{dx} = 1$ [Ans: $e^{y^{3/2}} \left(-\frac{1}{x} \right) = y^2 e^{y^{3/2}} - 2e^{y^{3/2}} + c$]
- 13) Solve: $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$ [Ans: $x^2 - y^2 = C(x^2 + y^2)^2$]
- 14) Solve: $\sqrt{1-y^2} dx = [\sin^{-1} y - x] dy$ [Ans: $x = (\sin^{-1} y - 1) + Ce^{\sin^{-1} y}$]
- 15) Solve the differential equation:
 $(x^2y + y^2x)dy = (x^3 + y^3)dx$ [Ans: $-\frac{y}{x} = \log(x-y) + c$]

Solve and Test your understanding on Differential equations

1. $\frac{dy}{dx} = \frac{x}{x^2 + 1}$

2. $(e^x + e^{-x}) \frac{dy}{dx} = (e^x - e^{-x})$

3. $\frac{dy}{dx} = \sin^3 x \cos^2 x + xe^x$

4. $e^{\frac{dy}{dx}} = x + 1; y(0) = 1$

5. $\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$

6. $(1 + x^2) \frac{dy}{dx} - x = 2 \tan^{-1} x$

7. $(x + 1) \frac{dy}{dx} = 2xy$

8. $\cos x(1 + \cos y) dx - \sin y(1 + \sin x) dy = 0$

9. $e^x \sqrt{1 - y^2} dx + \frac{y}{x} dy = 0$

10. $(1 + e^{2x}) dy + (1 + y^2) e^x dx = 0; x = 0, y = 0.$

11. $\frac{dy}{dx} (1 + y^2) (1 + \log x) dx + x dy = 0; x = 0, y = 0.$

12. $x(1 + y^2) dx - y(1 + x^2) = 0; x = 1, y = 0.$

13. $\frac{dy}{dx} = 1 + x + y + xy$

14. $y - x \frac{dy}{dx} = a \left(y^2 + \frac{dy}{dx} \right)$

15. $(x^2 - yx^2) dy + (y^2 + x^2 y^2) dx = 0$

RAPID FIRE ON DIFFERENTIAL EQUATIONS

$$y = \frac{dy}{dx} + \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$$

Q. 1. Find the order of the differential equation

Q. 2. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$.

Q. 3. Find the degree and order of the differential equations:

i. $(y''')^2 + (y'')^3 + (y')^4 \cdot y^5 = 0$

ii. $t^2 \frac{d^2s}{dt^2} - st \left(\frac{ds}{dt}\right)^2 = 5$

iii. $\left(\frac{d^3y}{dx^3}\right)^{2/3} = x$

iv. $y'' + 2y' + \sin y = 0$

v. $\left(\frac{d^2y}{dx^2}\right) + \cos\left(\frac{dy}{dx}\right) = 0$

vi. $\frac{d^2y}{dx^2} + 5x \left(\frac{dy}{dx}\right)^2 - 6y = \log x$

Q. 4. Find the solution of the differential equation $\frac{dy}{dx} + \frac{2y}{x} = 0$ with $y(1) = 1$.

Q. 5. Find the integrating factor of the differential equation :

i. $\frac{dy}{dx} + \frac{x + y \cos x}{1 + \sin x}$

ii. $(1 - y^2) \frac{dy}{dx} + yx = ay$ ($-1 < y < 1$)

iii. $x \frac{dy}{dx} - y = 2x^2$

iv. $\frac{dy}{dx} + 2y \tan x = \sin x$

ANSWERS

1. 1

2. $y = xc$

3. i. 2, 3
ii. 1, 2

1. 2, 3
2. 1, 2
3. not defined, 2

4. $\sqrt{y} = x$

5. i. $1 + \sin x$

ii. $\frac{1}{x}$

iii. $\frac{1}{\sqrt{1-y^2}}$

iv. $\sec^2 x$