

## DEFINITE INTEGRALS

### Definite Integral limit of sum

#### Problem No.1

Integrate the following definite as limit of sums:

$$\int_0^4 (x + e^{2x}) dx$$

**Solution:**

$$\text{We are given that } a = 0, b = 4 \quad h = \frac{4-0}{n} = \frac{4}{n}$$
$$nh = 4$$

By definition,

$$\int_a^b f(x).dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + \dots + f(a+(n-1)h)]$$

$$\int_0^4 (x + e^{2x}) dx = \lim_{h \rightarrow 0} h [f(0) + f(0+h) + f(0+2h) + \dots + f(0+(n-1)h)]$$

$$= \lim_{h \rightarrow 0} h [(0+e) + (h+e^{2h}) + (2h+e^{2(2h)}) + (3h+e^{3(2h)}) + \dots$$

$$+ \{(n-1)h + e^{(n-1)2h}\}]$$

$$= \lim_{h \rightarrow 0} h \left[ h[1+2+3+\dots+(n-1)] + \left( 1+e^{2h} + (e^{2h})^2 + (e^{2h})^3 + \dots + (e^{2h})^{n-1} \right) \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{h^2 n(n-1)}{2} + h \left( \frac{1 - (e^{2h})^n}{1 - e^{2h}} \right) \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{n^2 h^2 - nh^2}{2} - (1 - e^{2nh}) \frac{h}{1 - e^{2h}} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{4^2 - nh^2}{2} - (1 - e^8) \frac{1}{2 \times \frac{1 - e^{2h}}{2h}} \right]$$

$$\begin{aligned}
&= \frac{16}{2} - (1 - e^8) \lim_{h \rightarrow 0} \left( \frac{2h}{1 - e^{2h}} \right) \left( \frac{1}{2} \right) \\
&= \frac{16}{2} - (1 - e^8) \frac{1}{2} \\
&= \frac{16}{2} - \frac{1}{2} - \frac{e^8}{2} \\
&= \frac{15 - e^8}{2}
\end{aligned}$$

Evaluate the following Definite Integrals using limit of sum

$$1. \int_0^2 (3x^2 + 4) dx \quad 4. \int_0^3 (2x^2 - 5) dx.$$

$$2. \int_1^3 x^2 dx. \quad 5. \int_0^2 (x^2 + 3) dx.$$

$$3. \int_0^2 (x^2 + 2) dx. \quad 6. \int_0^4 (x + 7) dx.$$

### ANSWERS

$$1. 16 \text{ Sq. Units} \quad 4. \frac{26}{3} \text{ Sq. Units}$$

$$2. 3 \text{ Sq. Units} \quad 5. \frac{20}{3} \text{ Sq. Units}$$

$$3. \frac{26}{3} \text{ Sq. Units} \quad 6. 36 \text{ Sq. Units}$$

### Important Questions on Definite Integrals

1.  $\int_1^2 \left( \frac{x-1}{x^2} \right) e^x dx.$

2.  $\int_0^4 |x^2 - 4| dx.$

3.  $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx.$

4.  $\int_0^\pi \frac{x \tan x}{\sec x \operatorname{cosec} x} dx.$

5.  $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx.$

6.  $\int_{-5}^5 |x+2| dx.$

7.  $\int_0^{\pi/2} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx.$

8.  $\int_0^{\pi/2} \sin^3 x dx.$

9.  $\int_0^\pi \frac{x}{1 + \sin x} dx.$

10.  $\int_0^\pi \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx.$

11.  $\int_0^{\pi/2} \frac{dx}{1 + \sqrt{\cot x}}.$

12.  $\int_0^{\pi/2} \frac{\sin^2 \theta}{(1 + \cos \theta)^2} d\theta.$

### ANSWERS

1.  $\frac{e^2}{2} - e$

2.  $\frac{\pi}{4}$

3. 16

4.  $\frac{2}{3}$

5.  $\frac{\pi^2}{2}$

6.  $\pi$

7.  $\frac{\pi^2}{4}$

8.  $\frac{\pi^2}{2ab}$

9.  $\frac{\pi}{4}$

10.  $\frac{\pi}{4}$

11. 29

12.  $2 - \frac{\pi}{2}$