APPLICATION OF INTEGRALS

1) Find the area bounded by the curve y = 2 cosx and the x-axis from x = 0 to x = $x = 2\Pi$

Ans: 8 sq.units.

2) Find the area bounded by the x-axis part of the curve $y = 1 + \frac{8}{x^2}$ and the ordinates x = 2 and x = 4 If the ordinate at x = a divides the area into two equal parts find 'a'

Ans: Note 2<a<4 a = $2\sqrt{2}$

3) Find the area included between the parabolas $y^2=4ax$ and $x^2=4ay$ Ans: $16\frac{a^3}{3}$ sq.units.

4) Find the area of the segment cut off from the parabola $y^2=2x$ by the line y=4x-1

Ans: $\frac{9}{32}$ sq.units

5) Show that the area enclosed by the circle $x^2+y^2 = 64a^2$ and the parabola $y^2= 12ax$ is $a^2\left(\frac{16}{\sqrt{3}} + \frac{64\Pi}{\sqrt{3}}\right)$

6) Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area.

Ans: $\frac{5}{2}\left[\text{Sin}^{-1}\frac{2}{\sqrt{5}} + \text{Sin}^{-1}\frac{1}{\sqrt{5}}\right] - \frac{1}{2}$

7) Find the area of the region bounded by the curve C. y = tanx the tangent drawn to C at $x = \Pi/4$ and the x-axis. Ans: $\frac{1}{2}\left(\log 2 - \frac{1}{2}\right)$

8) Find the area of the region lying above x-axis and included between the curves $x^2+y^2 = 2ax$ and $y^2 = ax$ Ans: $a^2\left(\frac{\Pi}{4} - \frac{2}{3}\right)$

9) Sketch the region bounded by the curves $y = x^2$ and $y = \frac{2}{1+x^2}$ and find its area.

Ans: $\Pi - \frac{2}{3}$

10) Find the area of the smaller region bounded by the curve $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the straight line $\frac{x}{4} + \frac{y}{3} = 1$ Ans: $\frac{\Pi}{3}$ sq.units.

11) Using integration find the area of the triangle ABC where A is (2,3) B(4,7) and C(6,2)

Ans: 4 sq.units.

12) Using integration find the area of the triangle ABC whose vertices are A(3,0), B(4,6) and C(6,2)

Ans: 8 sq.units.

13) Find the area included between the curves $(x-1)^2+y^2 = 1$ and $x^2+y^2 = 1$

Ans: $\left(\frac{2\Pi}{3} - \frac{\sqrt{3}}{2}\right)$ sq.units.

14) Sketch the region common to the circle $x^2+y^2 = 25$ and the parabola $y^2 = 8x$ also find the area of the region using integration.

Ans : $\left\{\frac{\sqrt{2}}{3}\left(\sqrt{41}-4\right)^{3/2}+\frac{25}{4}\Pi-\frac{25}{2}\sin^{-1}\left(\frac{\sqrt{41}-4}{5}\right)\right\}$

15) Find the area of the circle $x^2 + y^2 = a^2$ Ans: $\Pi\, a^2$ sq.units.

16) Sketch the region of the ellipse and find its area using integration. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad a > b$

Ans: Πab sq.units.

17) Find the area of the region given by : $\{(x,y): x^2 \le y \le |x|\}$ Ans: $\frac{1}{3}$ sq.units

18) Find the area of the region

$\{(x,y): y^2 \le 4x, 4x^2+4y^2=9\}$ Ans: $\left\{\frac{\sqrt{2}}{6}+\frac{9\Pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}\right\}$ sq.units.

19) Find the area of the region bounded by the circle $x^2+y^2 = 16$ and the line $y = x$ in the first quadrant. Ans: 2Π sq.units.

20) Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$ Ans: $\frac{ab}{4}(\Pi-2)$ sq.units.

21) Find the area bounded by the curve $y = \sin x$, x=axis and between $x = 0$, $x = \Pi$

Ans: 2 sq.units.

22) Sketch the graph of $y = |x-1|$ and evaluate $\int_{-2}^{4}|x-1|dx$ Ans: 9 sq.units.

23) Find the area of the region enclosed between the circles $x^2+y^2 = 1$ and $\left(x-\frac{1}{2}\right)^2+y^2=1$

Ans: $\left(\frac{-2\sqrt{3}+\sqrt{15}}{16}-2\sin^{-1}\frac{1}{4}+\Pi\right)$ sq.units.

24) Draw the rough sketch of $y = \sin 2x$ and determine the area enclosed by the lines $x = \Pi/4$ and $x = \frac{3\Pi}{4}$

Ans: 1 sq.units.

25) Compute the area bounded by the lines $x+2y = 2$, $y-x = 1$ and $2x+y = 7$.

Ans: 6 sq.units.