

PRACTICE 3 – D GEOMETRY

- 1) Show that the line $\vec{r} = (2\vec{i} - 2\vec{j} + 3\vec{k}) + \lambda(\vec{i} - \vec{j} + 4\vec{k})$ is parallel to the plane $\vec{r} \cdot (\vec{i} + 5\vec{j} + \vec{k}) = 5$
- 2) Find the equation of the plane passing through the point (1, 4, -2) and parallel to the plane $-2x + y - 3z = 7$
[Ans: $2x - y + 3z + 8 = 0$]
- 3) What are the conditions that the planes $a_1x + b_1y + c_1z = d_1$ & $a_2x + b_2y + c_2z = d_2$ are
(i) parallel (ii) perpendicular to each other?

Ans: (i) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (ii) $a_1a_2 + b_1b_2 + c_1c_2 = 0$
- 4) Find the value of k for which the two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = z$ intersect at a point?

[Ans: $k = \frac{9}{2}$]
- 5) Find the equation of the plane passing through the point (1, 6, 3) and perpendicular to the plane $2x + 3y - z = 7$
(Ans: $3x + y + 9z = 36$)
- 6) Find the value of k for which the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are co-planar.
[Ans: $k = 0, -3$]
- 7) Find the equation of the plane passing through the point (1, 1, 1) and perpendicular to each of the plane $x + 2y + 3z = 7$ and $2x - 3y + 4z = 0$
[Ans: $17x + 2y - 7z - 12 = 0$]
- 8) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = 2$ intersect. Also find the point of intersection.
[Ans: $(-1, -1, -1)$]
- 9) Find the image of the point (1, 2, 3) in the plane $x + 2y + 4z = 38$. [Ans: $(3, 6, 11)$]
- 10) Find the equation of the plane passing through the points (1, -1, 2) and (2, -2, 2) and perpendicular to the plane $6x - 2y + 2z = 9$.
[Ans: $x + y - 2z + 4 = 0$]
- 11) Find the foot of the perpendicular drawn from the point A(1, 0, 3) to the join of the points B(4, 7, 1) and C(3, 5, 3)

[Ans: $\frac{5}{3}, \frac{7}{3}, \frac{17}{3}$]
- 12) Find the length and co-ordinates of the foot of perpendicular from point (1, 1, 2) to the plane $2x - 2y + 4z + 5 = 0$

[Ans: $\frac{13\sqrt{6}}{12}, \left(-\frac{1}{12}, \frac{25}{12}, -\frac{1}{6}\right)$]
- 13) Find the equation of the plane through the points (-1, 1, 1) and (1, -1, 1) perpendicular to the plane $x + 2y + 2z = 5$
(Ans: $2x + 2y - 3z + 3 = 0$)
- 14) Find the perpendicular distance of point (2, 3, 4) from the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$

[Ans: $\frac{1}{49}\sqrt{44541}$]
- 15) The foot of the perpendicular drawn from the origin to the plane is (2, 5, 7). Find the equation of the plane.
[Ans: $2x + 5y + 7z = 78$]

- 16) Find the values of P so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. [Ans: $\frac{70}{11}$]
- 17) Find the shortest distance between two lines whose vector equations are $\vec{r} = (1-t)\vec{i} + (t-2)\vec{j} + (3-2t)\vec{k}$ and $\vec{r} = (s+1)\vec{i} + (2s-1)\vec{j} + (2s+1)\vec{k}$ [Ans: $\frac{8}{\sqrt{29}}$]
- 18) Find the vector equation of the plane through the intersection of the planes $\vec{r} \cdot (2\vec{j} + 6\vec{j}) + 12 = 0$ & $\vec{r} \cdot (3\vec{i} - \vec{j} + 4\vec{k}) = 0$ which are at a unit distance from the origin. [Ans: $\vec{r} \cdot (-\vec{i} + 2\vec{j} - 2\vec{k}) + 3 = 0$]
- 19) Find the equation of the line passing through the point (3,0,1) and parallel to the planes $x+2y = 0$ and $3y - z = 0$ [Ans: $\vec{r} = (3\vec{i} + \vec{k}) + \lambda(-2\vec{i} + \vec{j} + 3\vec{k})$]
- 20) Find the reflection of the point (1,2,-1) in the plane $3x-5y+4z = 5$ [Ans: $(\frac{73}{25}, \frac{-6}{5}, \frac{39}{25})$]
- 21) Find the distance of the point (1,-2,3) from the plane $x-y+z = 5$ measured parallel to the line $\frac{x+1}{2} = \frac{y+3}{2} = \frac{z+1}{-6}$ [Ans: 1]
- 22) Find the distance of the point (2,3,4) from the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$ measured parallel to the plane $3x+2y+2z+5 = 0$

RAPID FIRE ON 3 – D GEOMETRY

- Q. 1. Find the direction ratios of the line $6x - 1 = 2y + 3 = 5 - z$.
- Q. 2. Find the direction cosines of a line making angles $60^\circ, 90^\circ, 30^\circ$ with coordinate axes.
- Q. 3. Write the direction ratios of the line normal to the plane $x + 2y - 3z + 4 = 0$.
- Q. 4. Find the direction cosines of y-axis.
- Q. 5. Find the equation of a plane making equal intercepts on axes and passing through the point $(2, 8, 3)$.
- Q. 6. Find the length of the perpendicular from the point $(2, 3, 7)$ to the plane $3x - y - z = 7$.
- Q. 7. Write the Cartesian equations of the line passing through $(-1, 2, 3)$ and equally inclined to the positive direction of x-axis.
- Q. 8. Find the angle between the planes $2x - 3y + 4z = 1$ and $-x + y = 4$.
- Q. 9.
If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular. Find the value of k.
- Q. 10. Find the angle between the line $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z-2}{4}$ and the plane $2x + y - 3z + 4 = 0$.
- Q. 11. Find the distance of the point $2\hat{i} + \hat{j} - \hat{k}$ from the plane $\vec{r} \cdot (\hat{i} - 2\hat{j} + 4\hat{k}) = 9$.
- Q. 12. Find the angle between the following pair of lines:
 $\frac{x+4}{3} = \frac{y-1}{5} = \frac{z+3}{4}, \frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.
- Q. 13. Direction ratios of a line are proportional to $(1, -3, 2)$, then find its direction cosines.
- Q. 14. Write the equation of the plane whose intercepts on the coordinate axes are $-4, 2, 3$.
- Q. 15. Find the intercepts cut off by the plane $2x + y - z = 5$.

ANSWERS

1. $\langle 1, 3, -6 \rangle$

2. $\left\langle \frac{1}{2}, 0, \frac{\sqrt{3}}{2} \right\rangle$

3. $\langle 1, 2, -3 \rangle$

4. $\langle 0, 1, 0 \rangle$

5. $x + y + z - 13 = 0$

6. $\sqrt{11}$

7. $x + 1 = y - 2 = z - 3$

8. $\cos^{-1} \left(\frac{-5}{\sqrt{58}} \right)$

9. $-\frac{10}{7}$

10. $\sin^{-1} \left(\frac{4}{\sqrt{406}} \right)$

11. $\frac{13}{\sqrt{21}}$

12. $\cos^{-1} \left(\frac{8}{5\sqrt{3}} \right)$

13. $\left\langle \frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}} \right\rangle$

14. $3x - 6y - 4z + 12 = 0$

15. $\frac{5}{2}, 5, -5$